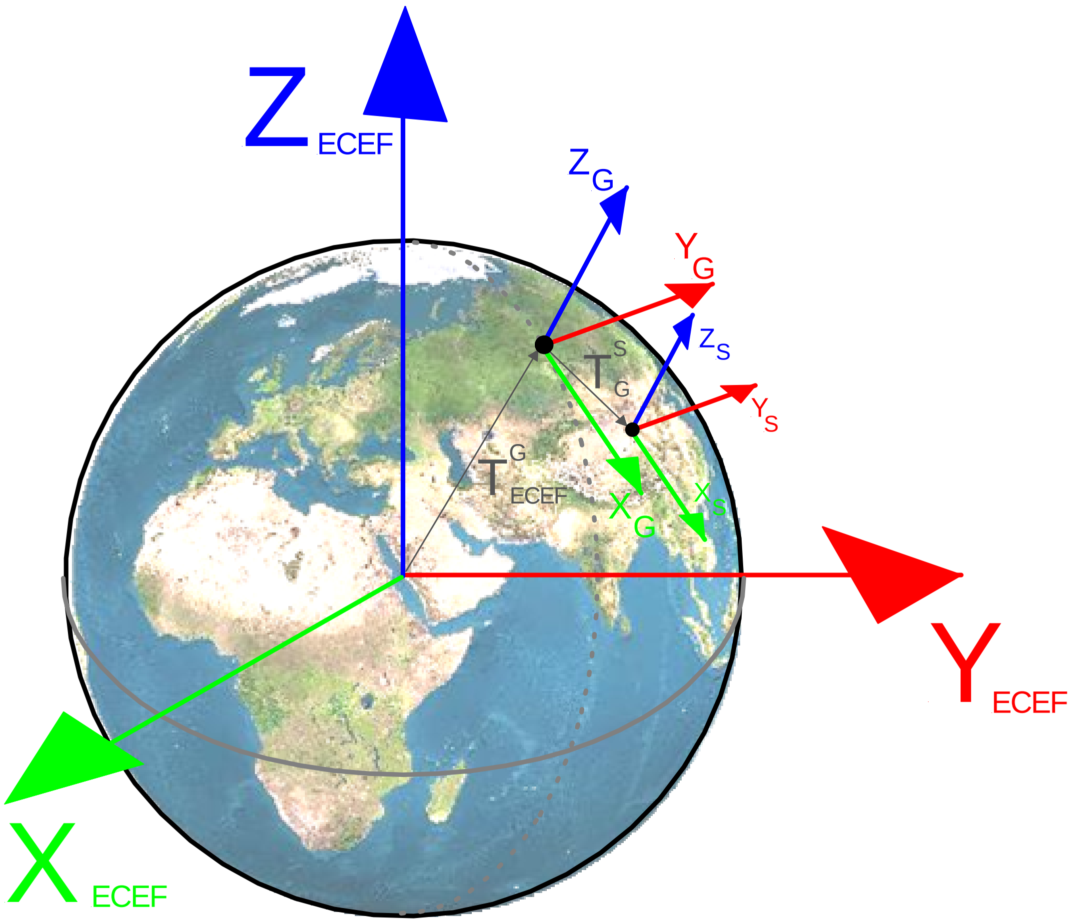
IREDES International Rock Excavation Data Exchange Standard

Coordinate System Description



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Document Information

IREDES Coordinate Systems Description

This document is part of the IREDES standard set of documents.

It is assumed that the reader of this document is familiar with:

- The purpose and the general layout of the IREDES standard.
- Basic usage of XML and XML schemas
- Basic coordinate system handling

Introduction into the IREDES standard is available in the IREDES Architecture description (IRArchit.pdf).

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2002-04	Initial document setup
2004-12	Layout changes and integration into IREDES document series

Contents

1 Introduction

There are many different coordinate systems, based on a variety of geodetic datums, units, projections, and reference systems in use today. For a tunnel a geodetic datum may be used, for a mine the coordinate system may be oriented along the ore. Some uses right handed systems, other left handed systems. This document describes how coordinate systems can be related and how to transfer the parameters for coordinate systems between different kinds of equipment.
All coordinate systems described in this document are orthogonal Cartesian systems.

2 Coordinate system overview

Geodetic datums define the reference systems that describe the size and shape of the earth. Hundreds of different datums have been used to frame position descriptions since Aristotle made the first estimates of the earth's size. Datums have evolved from those describing a spherical earth to ellipsoidal models derived from years of satellite measurements.

Modern geodetic datums range from flat-earth models used for plane surveying to complex models used for international applications, which completely describe the size, shape, orientation, gravity field, and angular velocity of the earth.

Referencing geodetic coordinates to the wrong datum can result in position errors of hundreds of meters. Different nations and agencies use different datums as the basis for coordinate systems used to identify positions in geographic information systems, precise positioning systems, and navigation systems. The diversity of datums in use today and the technological advancements that have made possible global positioning measurements with sub-meter accuracies requires careful datum selection and careful conversion between coordinates in different datums.

3 Naming convention of coordinate systems in the IREDES standard

The IREDES standard use the following coordinate system names:

ECEF	-	Earth Centred, Earth Fixed coordinate system has its origo in the centre of earth.
Global	-	Map datum used in an area, for instance Europe, South Africa, Canada.
Local	-	Used in a localised area, for instance a mine or a tunnel project
Project	-	Sub level in a localised area, for instance a mine can have different coordinate systems for different ores.
Site	-	Where the drill rig is working

Drill plan - Reference system for a drill plan set up independently from any site coordinate system

3.1 Linked list of coordinate systems

Except for *ECEF* and *Site* the naming here is given for convenience and to establish common terms, since coordinate systems in the IREDES standard are defined as a linked list of coordinate systems. This implies that the user can choose to use as many levels of coordinate systems as necessary. In many cases the *Global*, *Local* and *Project* coordinate systems are one and the same.
There is also a "one to many" option in this. In a mine there can be for example one *Local* and three *Project* coordinate systems in use.

3.2 General coordinate system definition

A coordinate system is defined by its transformation into the higher order coordinate system. Since left and right-handed systems are in use, it is necessary with a parameter for this. A coordinate system will then have a description as follow:

$$\mathrm{T}_t^f = \begin{bmatrix} a_{11} & a_{12} & a_{13} & X_0 \\ a_{21} & a_{22} & a_{23} & Y_0 \\ a_{31} & a_{32} & a_{33} & Z_0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, (L/R)$$

- *f* is the "from" and *t* is the "to" coordinate system, or lower and higher order coordinate systems
- a_{11} to a_{33} is the orientation matrix . $\begin{bmatrix} a_{11} \\ a_{21} \\ a_{31} \end{bmatrix}$, $\begin{bmatrix} a_{12} \\ a_{22} \\ a_{32} \end{bmatrix}$ and $\begin{bmatrix} a_{13} \\ a_{23} \\ a_{33} \end{bmatrix}$ gives the direction of the X-, Y- and Z-vectors for the "from" coordinate system in the "to" coordinate system
- X_0, Y_0 and Z_0 describe the vector between the origos for the two coordinate systems given in the "to" coordinate system.
- *L/R* is L for left handed and R for right handed top most coordinate system in the linked list. For all other coordinate systems this parameter is 0 since the transformation tells if the coordinate system is left handed or right handed.

3.2.1 Naming of coordinate system transformations

The transformations between the defined coordinate systems are written:

T_{ECEF}^{G} Transformation from *Global* to *ECEF*

T_{G}^{L} Transformation from *Local* to *Global*

T_{L}^{P} Transformation from *Project* to *Local*

T_{L}^{Pn} Transformation from *Project*$_n$ to *Local*

T_{P}^{Sn} Transformation from *Site$_n$* to *Project*

T_{S}^{D} Transformation from *Drill plan* to *Site*

3.2.2 Complete transformations for linked lists

Some examples of complete transformations are shown below:

$\mathrm{T}_{ECEF}^{S_1} = \mathrm{T}_{ECEF}^{G} \bullet \mathrm{T}_{G}^{L} \bullet \mathrm{T}_{L}^{P} \bullet \mathrm{T}_{P}^{S_1}$ Transformation from *Site$_1$* to *ECEF*

$\mathrm{T}_{L}^{S_{11}} = \mathrm{T}_{L}^{P_1} \bullet \mathrm{T}_{P_1}^{S_1}$ Transformation from *Site$_1$* in *Project$_1$* to *Local*

$\mathrm{T}_{L}^{S_{21}} = \mathrm{T}_{L}^{P_2} \bullet \mathrm{T}_{P_2}^{S_1}$ Transformation from *Site$_1$* in *Project$_2$* to *Local*

3.2.3 Top most coordinate system

Since we need to know if the top most coordinate system is left handed or right handed, we also have to define this. The 3x3 rotation part of the transformation is for the top most coordinate system equal to the Identity matrix. The vector between the origos is the zero vector. Two examples are:

ECEF: $$\mathrm{T} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, (R)$$

Left-handed local system: $$\mathrm{T} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, (L)$$

3.3 *Earth Centred, Earth Fixed Cartesian coordinates*

This coordinate system has been chosen to be the top-level coordinate system in the IREDES standard. However, it is not necessary to use this system unless global positioning equipment is in use.

Earth centred, earth-fixed Cartesian coordinate system is defined as follow (See figure 1):

- Earth centred, earth-fixed, X, Y, and Z, Cartesian coordinates (XYZ) define three-dimensional positions with respect to the centre of mass of the reference ellipsoid.
- The Z-axis points toward the North Pole.
- The X-axis is defined by the intersection of the plane define by the prime meridian and the equatorial plane.

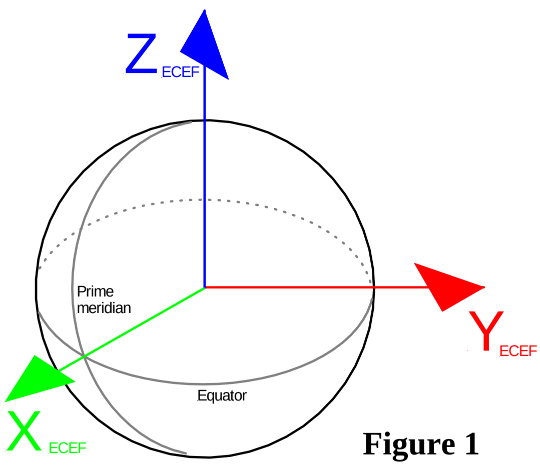
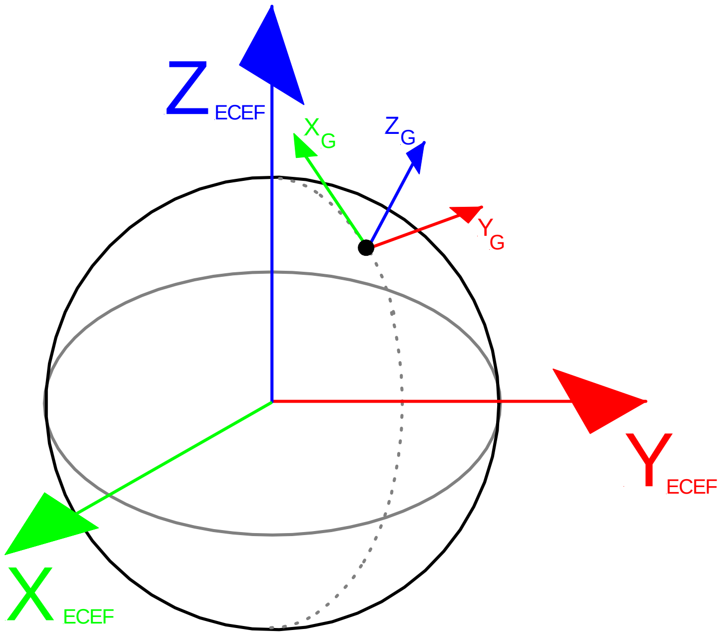


Figure 1

- The Y-axis completes a right-handed orthogonal system by a plane, 90 degrees east of the X-axis and its intersection with the equator.

3.4 Global coordinate system

This can for instance be a national grid system, which are commonly used in tunnelling. The coordinate system can be right or left-handed. A left handed example is coordinate system “G”, as shown in figure 2. A more detailed description is found in the appendix “THE FINNISH NATIONAL GRID COORDINATE SYSTEM”



3.5 Local and project coordinate systems

The *local* coordinate system can be the main coordinate system for a mine or a tunnel project and there are possible to use several *project* coordinate systems defined in this coordinate system. Typically the mine uses the *project* coordinate systems to define local coordinate systems for each ore. For a tunnelling project the *local* coordinate system may be the same as the *global* coordinate system or a system where the most

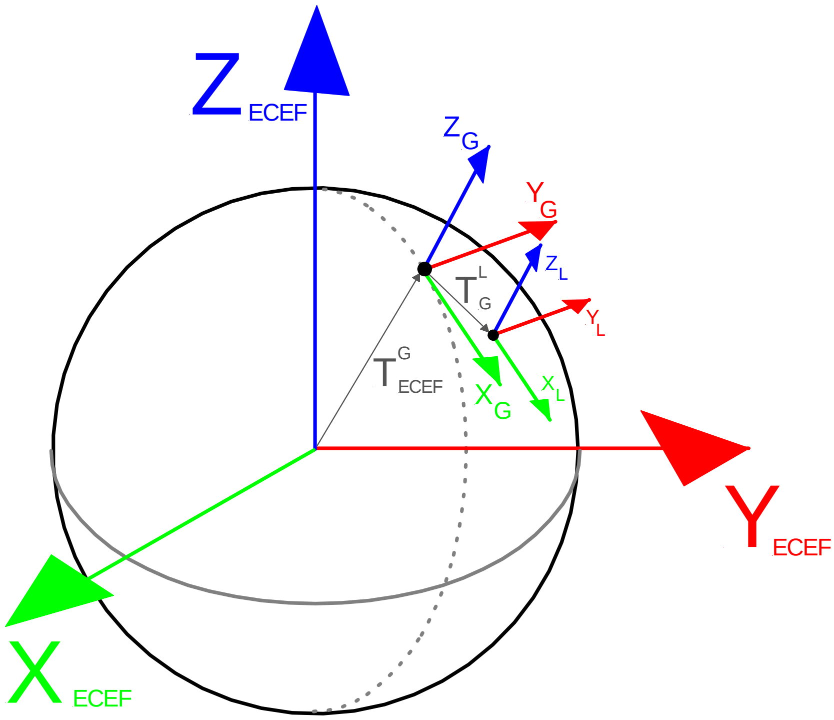


Figure 1

significant digits are removed to simplify map numbers. A right-handed example is coordinate system “L”, as shown in figure 3.

3.6 Site coordinate system

These coordinate systems are the places where the rig is working. They can be transferred to the drill rig or calculated by the rig based on additional information as the tunnel line. The two alternatives are described in chapter 3.6.1 and 3.6.2.

3.6.1 List of coordinate systems

The first alternative is a list of coordinate systems as shown in figure 4. They all refer to the *project* coordinate system. They can be defined very close or the rig has the possibility to interpolate in between two coordinate systems.

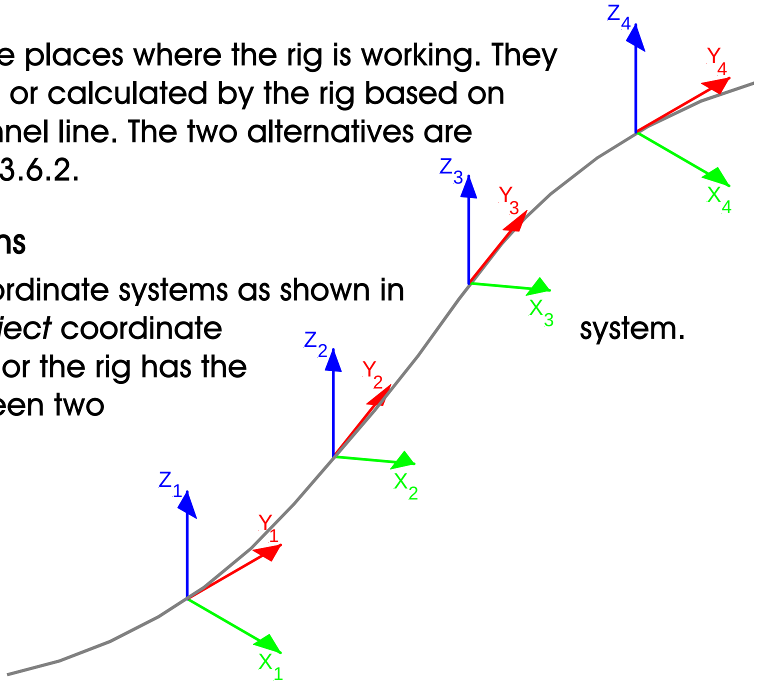


Figure 2

3.6.2 Tunnel line

The second alternative is a tunnel line, which is a list of points given in *project* coordinate system. In addition to X, Y and Z values, each point has given a peg value and an inclination angle.

3.6.2.1 Peg value

The peg value is the distance from the beginning of the line to a point,
measured along the line. A special start point is defined as the start of the tunnel.

3.6.2.2 Inclination angle

The inclination angle is the rotation angle around the tunnel line and defined so that seen along the line in increasing peg values a clockwise rotation is positive. Zero degrees means that the XY-plane in the drill plan coordinate system is parallel to the XY-plane of the *project* coordinate system.

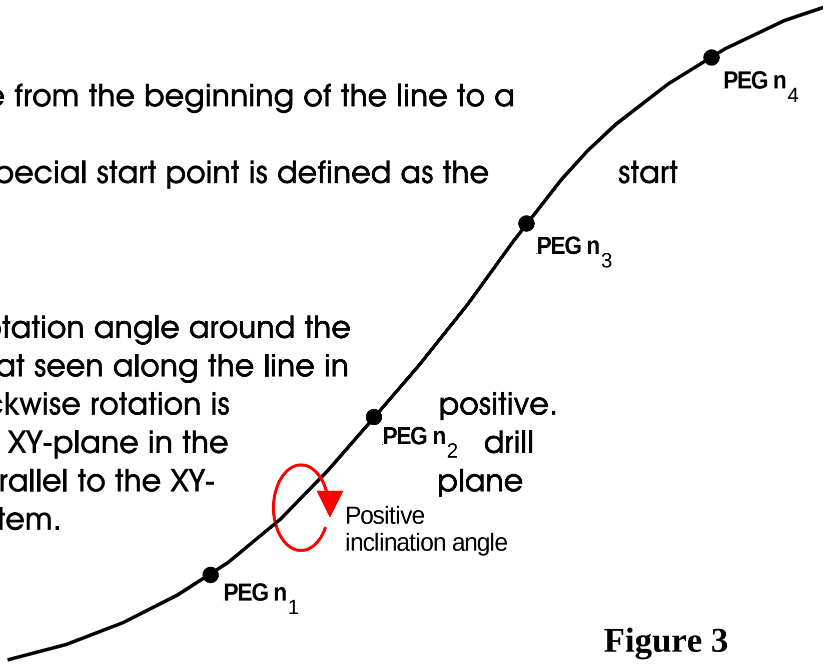


Figure 3

3.7 Drill plan coordinate system

This is a reference system used for defining the holes in the drill plan. Its origo is defined as the intersection between the left side/wall and the base without any ditches. See figure 6. Please, be aware that the angle between the base of the drill plan and the X axis is not the inclination angle, but an angle designed into the plan. The transformation from *Drill plan* to *Site* must follow the drill plan. When reusing a drill plan from one site to another site this transformation (T_S^D) may have to be redefined.

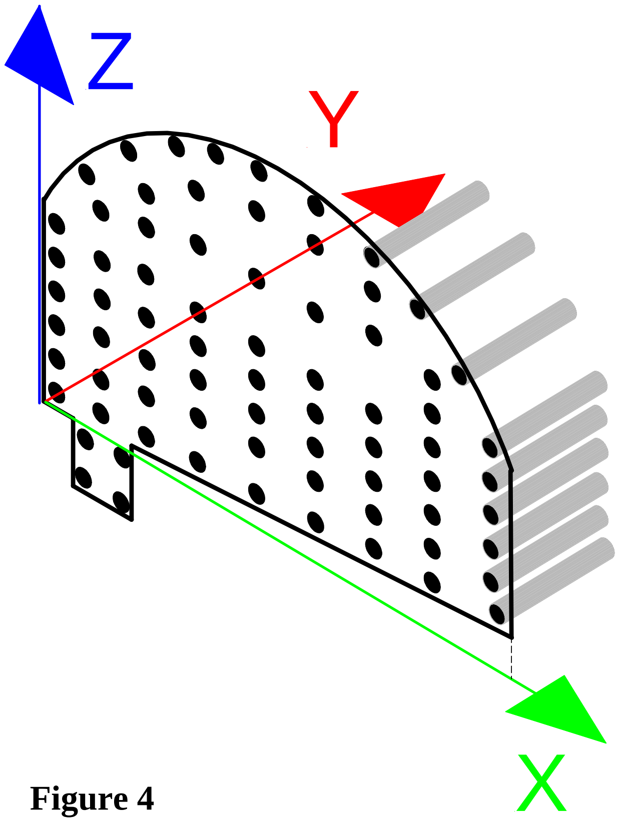


Figure 4

3.8 How to decide if a system is left- or right-handed

To decide if a system is left-handed or right-handed can be decided by the following equation, using the definition defined in chapter 3.2:

$$T_t^f = \begin{bmatrix} a_{11} & a_{12} & a_{13} & X_0 \\ a_{21} & a_{22} & a_{23} & Y_0 \\ a_{31} & a_{32} & a_{33} & Z_0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, (L/R)$$

If we define $n_x = \begin{bmatrix} a_{11} \\ a_{21} \\ a_{31} \end{bmatrix}$, $n_y = \begin{bmatrix} a_{12} \\ a_{22} \\ a_{32} \end{bmatrix}$ $n_z = \begin{bmatrix} a_{13} \\ a_{23} \\ a_{33} \end{bmatrix}$ and $p = (n_x \times n_y) \bullet n_z$ then we get a change from left-handed to right-handed or from right-handed to left-handed if p < 0. Since we know the type of the top most coordinate system, we can decide which type the lower level systems have.

4 Laser description

The laser is defined with a start vector and an end vector, given in the lowest level coordinate system transferred to the rig.

5 Tunnel line description

A tunnel line is defined with a set of vectors and an inclination angle for each vector. The vectors are given in the lowest level coordinate system transferred to the rig.

The positive direction of the line is defined as positive from point one towards the last point. Positive inclination angle is defined as the clockwise rotation when looking in the positive direction of the line.
When the inclination angle is zero, the X-axis for the site coordinate system is parallel to the horizontal plane and the Z-axis of a right-handed site system is pointing up in the lowest level coordinate system transferred to the rig.

6 Object oriented descriptions (XML)

The XML description is shown in figure 7, 8 and 9 below.

6.1 Coordinate systems

Row 4 of the transformation is omitted.

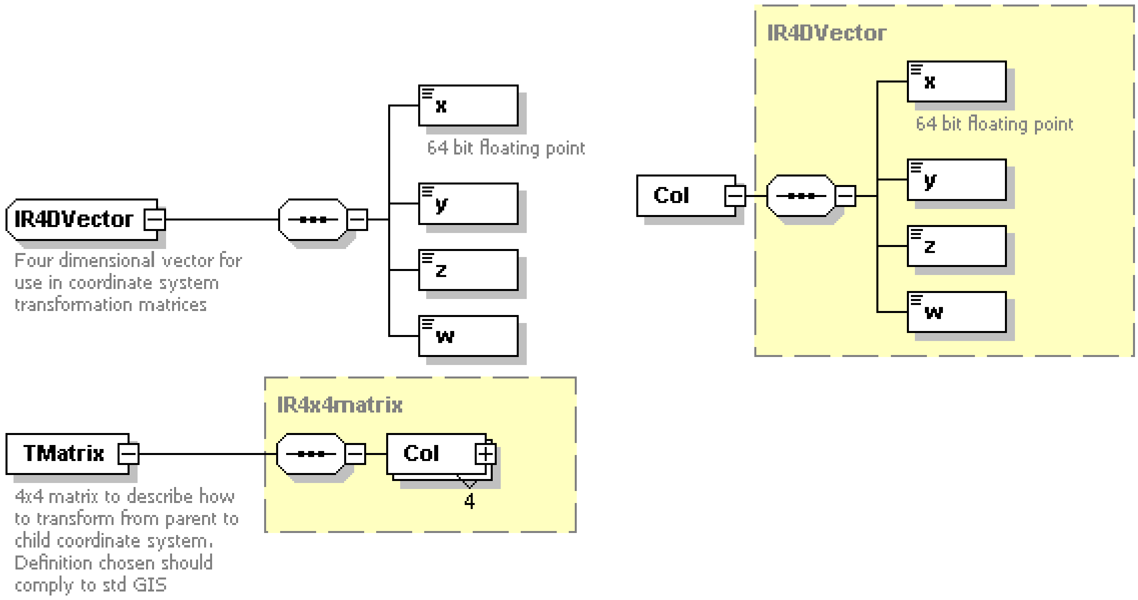


Figure 7a: Single 4-Dimensional Vector component as basic elements of the transformation matrix

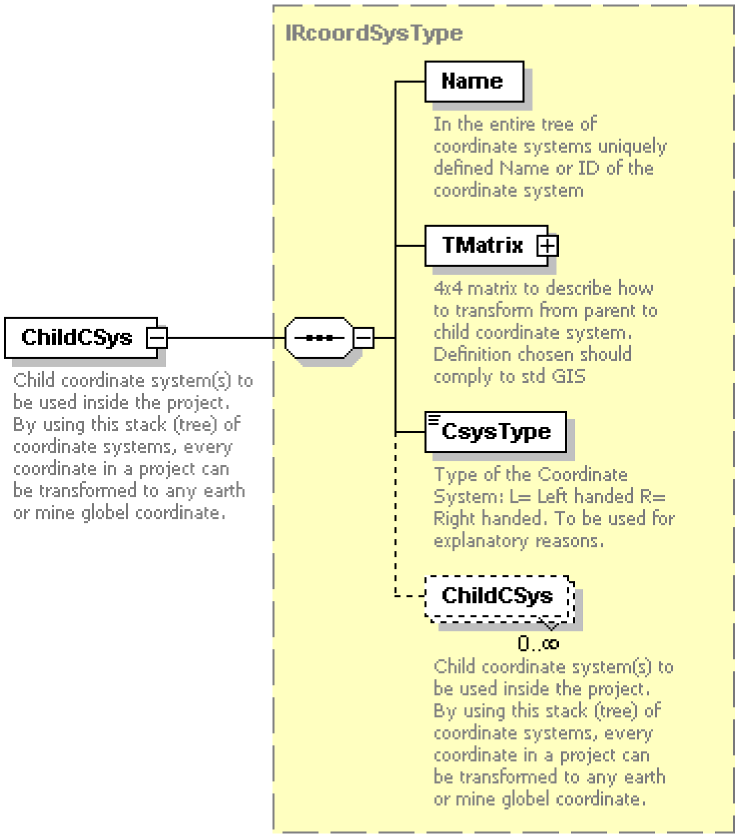


Figure 5b: Definition of one coordinate system, the transformation matrix to the parent and the link to the next lower level coordinate system(s).

6.2 Tunnel Line

Not finished

Figure 6

6.3 Laser Line

Not finished

Figure 7

7 Examples

7.1 Tunnel construction

In this example we have a tunnel line pointing in North North-East direction with the following parameters:
Direction: 22.5°
Inclination: –2°
Steepness: 5° and increasing (uphill)
Site position: North 17082m, West 23096m, Up 44m

The tunnel line intersects with the drill plan in upper right corner, 10 meters to the right and 3 meters up.

A global left-handed map system is used, X points North, Y points East and Z points up.

The site coordinate system is right-handed and has its Y-axis parallel with the tunnel line.

See figure 10.

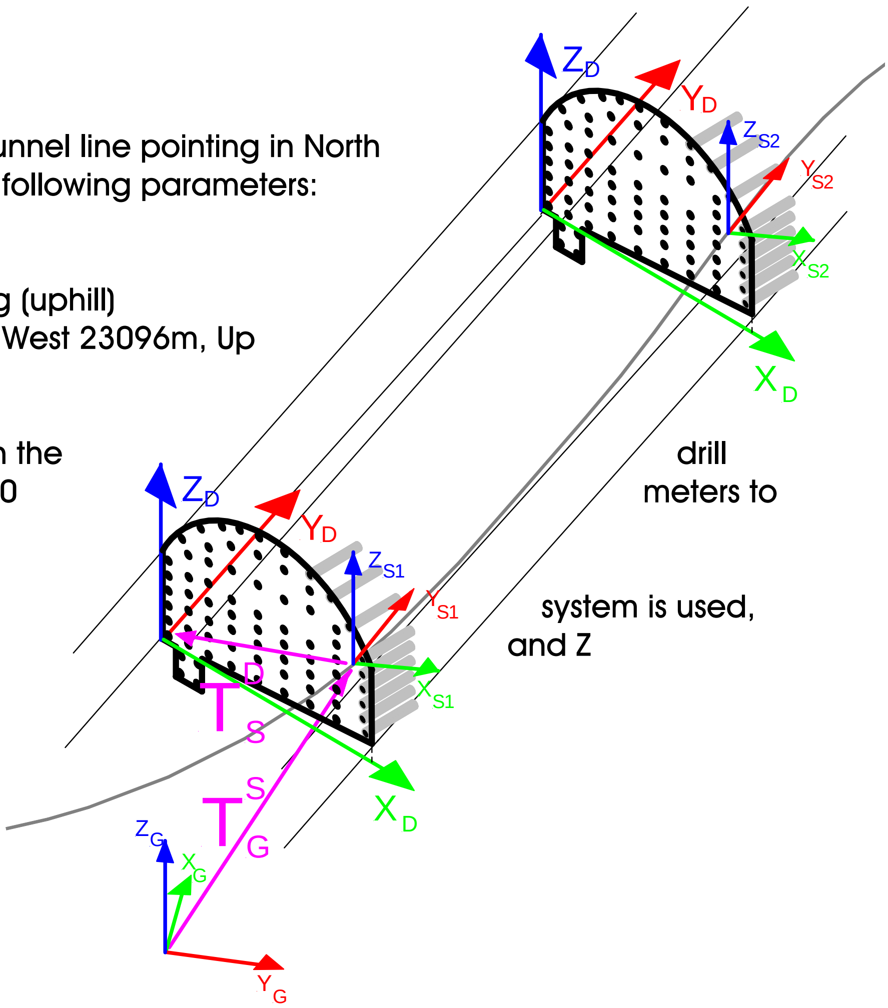


Figure 8

The different transformations becomes as follow:

Top most left-handed global system: $\mathrm{T} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, (L)$

Transformation from site to global system:

$$\mathrm{T}_G^S = \begin{bmatrix} 1 & 0 & 0 & 17082 \\ 0 & -1 & 0 & -23096 \\ 0 & 0 & 1 & 44 \\ 0 & 0 & 0 & 1 \end{bmatrix} \bullet \begin{bmatrix} \cos(-90-22.5) & -\sin(-90-22.5) & 0 & 0 \\ \sin(-90-22.5) & \cos(-90-22.5) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \bullet \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(5) & -\sin(5) & 0 \\ 0 & \sin(5) & \cos(5) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \bullet \begin{bmatrix} \cos(-2) & 0 & \sin(-2) & 0 \\ 0 & 1 & 0 & 0 \\ -\sin(-2) & 0 & \cos(-2) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, (0)$$

The sub transformations are from left to right, transformation from left handed to right handed system located in site origo, rotation so that Y-axis points North North-East, rotation for steepness and rotation for inclination.

Transformation from drill plan to site: $T_S^D = \begin{bmatrix} 1 & 0 & 0 & -10 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 1 \end{bmatrix}, (0)$

A hole has the following data in the drill plan
Start vector: 5m, 0m, 4m
End vector: 5m, 5m, 4.3m

Its coordinates in the global system becomes:

Start vector: $V_{S_G} = T_G^S \bullet T_S^D \bullet \begin{bmatrix} 5 \\ 0 \\ 4 \\ 1 \end{bmatrix}$

End vector: $V_{E_G} = T_G^S \bullet T_S^D \bullet \begin{bmatrix} 5 \\ 5 \\ 4.3 \\ 1 \end{bmatrix}$

7.2 Mine

This mine uses a right-handed local coordinate system. X points towards North, Y points East and Z points down.

The mine has several project coordinate systems. The one in this example is rotated –24° around the Z_L-axis. Its origo is 300m south, 500m east and 400m down with respect to the local system.

The project has two parallel tunnels 20m apart, where the

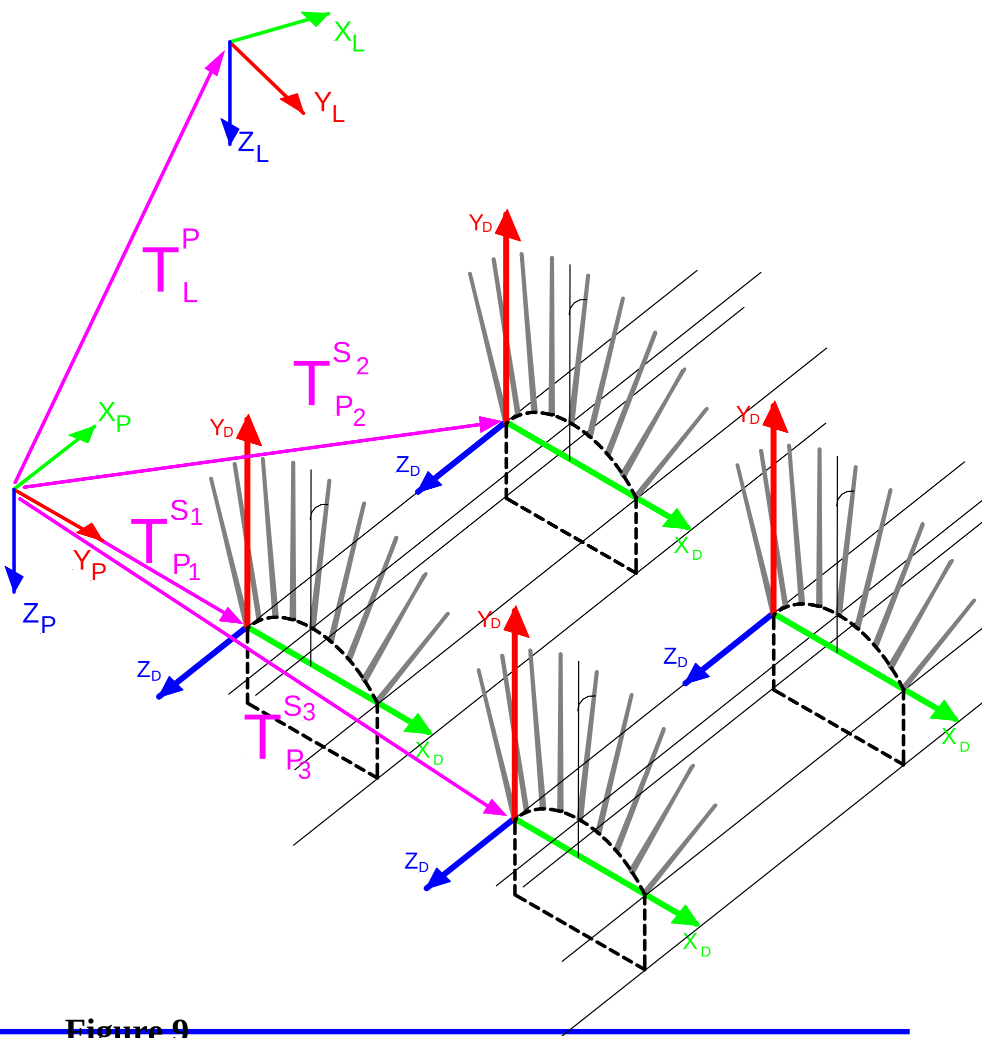


Figure 9

production drilling is done. The spacing along the tunnels is 8m and origo for site 1 has X_P=0 and Y_P=20m. While site 3 has X_P= -4m.

Site systems are parallel to the project coordinate system and identical to the drill plan systems. This gives all drill plan to site transformations equal to the identity matrix: $T_S^D = I, (0)$

We can the set up the following transformations:

Top most right-handed local system: $T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, (R)$

Transformation from project to local: $T_L^P = \begin{bmatrix} \cos(-24) & -\sin(-24) & 0 & -300 \\ \sin(-24) & \cos(-24) & 0 & 500 \\ 0 & 0 & 1 & 400 \\ 0 & 0 & 0 & 1 \end{bmatrix}, (0)$

Transformation from site 1 to project: $T_P^{S_1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 20 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, (0)$

Transformation from site 2 to project: $T_P^{S_2} = \begin{bmatrix} 1 & 0 & 0 & 8 \\ 0 & 1 & 0 & 20 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, (0)$

Transformation from site 3 to project: $T_P^{S_3} = \begin{bmatrix} 1 & 0 & 0 & -4 \\ 0 & 1 & 0 & 40 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, (0)$

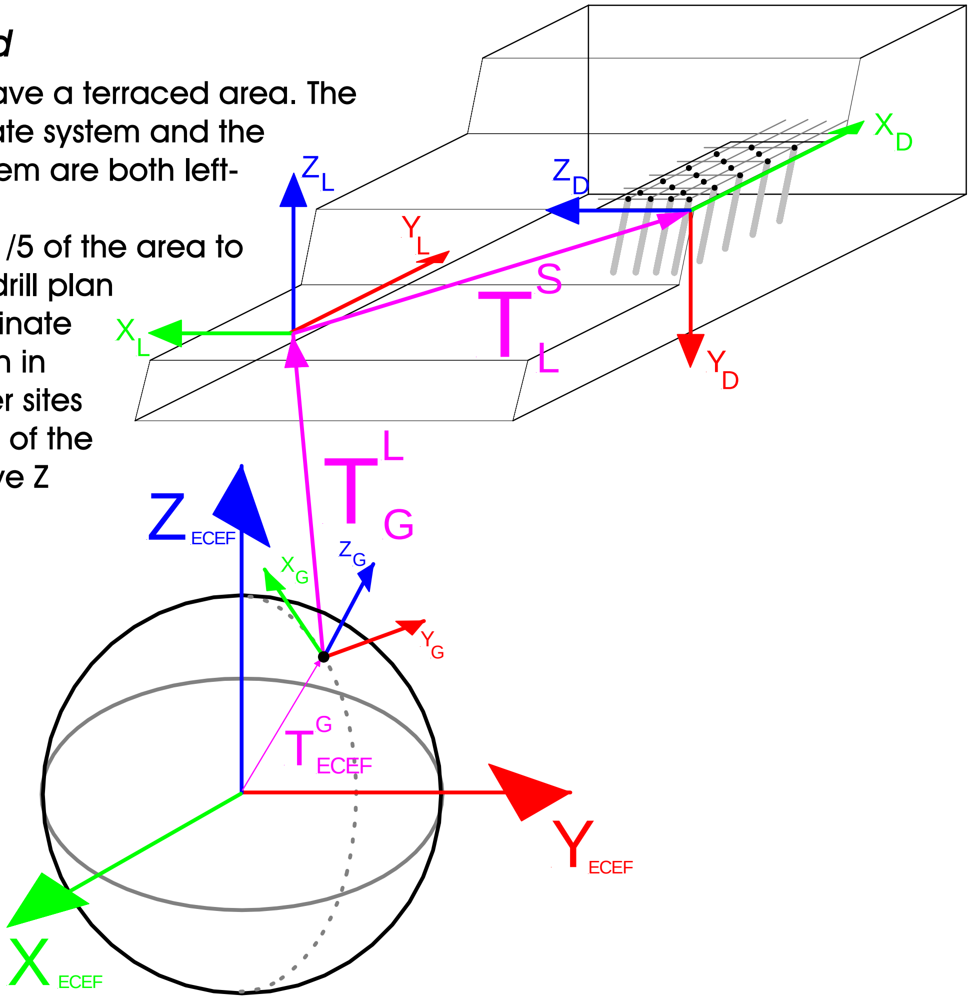
Transformation from drill plan to site$_n$: $T_{S_n}^D = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, (0)$

Then the complete transformation from drill plan to local coordinate systems becomes:

$$T_{\mathrm{L}}^{D} = T_{\mathrm{L}}^{P} \bullet T_{\mathrm{P}}^{S_n} \bullet T_{\mathrm{Sn}}^{D} = T_{\mathrm{L}}^{P} \bullet T_{\mathrm{P}}^{S_n}$$

7.3 Above ground

In this example we have a terraced area. The global map coordinate system and the local coordinate system are both left-handed.
The drill plan covers 1/5 of the area to be drilled. Sites and drill plan have identical coordinate system. Site 1 is shown in the drawing, the other sites are shifted the height of the drill plan in the positive Z direction.



Origo of the global coordinate system is located at NAD-83 30:16:28.82N and 97:44:25.19W and in ECEF coordinates this becomes:
X = -742507.1m
Y = –5462738.5m
Z = 3196706.5m

The local coordinate system is rotated 143° around the Z-axis and its origo is 1200m north, 13000m east and 200m up with respect to the global system.

The local coordinate system is rotated 143° around the axis and its origo is 1200m north and 13000m east with respect to the global system. **Figure 10** Z-

The origo of the coordinate system for site 1 has the following values in the local system:
X = -50m, Y = 40m, Z = 30m, axis are parallel with axis for the local system as shown in figure 12. The height of the drill plan is 11m.
Site systems are identical to the drill plan systems. This gives all drill plan to site transformations equal to the identity matrix: $T_S^D = I, (0)$

We can the set up the following transformations:

Top most right-handed ECEF system: $T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, (R)$

Transformation from global to ECEF
30:16:28.82N represent a rotation about Z-axis of –97.74° and 97:44:25.19W represents a rotation about a Y-axis of –120.27°.

$$\mathrm{T}_{ECEF}^{G} = \begin{bmatrix} \cos(-97.74) & -\sin(-97.74) & 0 & -742507.1 \\ \sin(-97.74) & \cos(-97.74) & 0 & -5462738.5 \\ 0 & 0 & 1 & 3196706.5 \\ 0 & 0 & 0 & 1 \end{bmatrix} \bullet \begin{bmatrix} \cos(-120.27) & -\sin(-120.27) & 0 & 0 \\ \sin(-120.27) & \cos(-120.27) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \bullet \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},(0)$$

Transformation from local to global: $\mathrm{T}_{G}^{L} = \begin{bmatrix} \cos(-143) & -\sin(-143) & 0 & 1200 \\ \sin(-143) & \cos(-143) & 0 & 13000 \\ 0 & 0 & 1 & 200 \\ 0 & 0 & 0 & 1 \end{bmatrix},(0)$

Transformation from site 1 to local: $\mathrm{T}_{L}^{S_1} = \begin{bmatrix} 0 & 0 & 1 & -50 \\ 1 & 0 & 0 & 40 \\ 0 & -1 & 0 & 30 \\ 0 & 0 & 0 & 1 \end{bmatrix},(0)$

Transformation from site 2 to project: $\mathrm{T}_{P}^{S_2} = \begin{bmatrix} 0 & 0 & 1 & -39 \\ 1 & 0 & 0 & 40 \\ 0 & -1 & 0 & 30 \\ 0 & 0 & 0 & 1 \end{bmatrix},(0)$

Transformation from site 3 to project: $\mathrm{T}_{P}^{S_3} = \begin{bmatrix} 0 & 0 & 1 & -28 \\ 1 & 0 & 0 & 40 \\ 0 & -1 & 0 & 30 \\ 0 & 0 & 0 & 1 \end{bmatrix},(0)$

Transformation from drill plan to site$_n$: $\mathrm{T}_{S_n}^{D} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},(0)$

Then the complete transformation from drill plan to ECEF coordinate systems becomes:

$$\mathrm{T}_{\mathrm{ECEF}}^{D} = \mathrm{T}_{\mathrm{ECEF}}^{G} \bullet \mathrm{T}_{\mathrm{G}}^{L} \bullet \mathrm{T}_{\mathrm{L}}^{S_n} \bullet \mathrm{T}_{\mathrm{Sn}}^{D} = \mathrm{T}_{\mathrm{ECEF}}^{G} \bullet \mathrm{T}_{\mathrm{G}}^{L} \bullet \mathrm{T}_{\mathrm{L}}^{S_n}$$

8 Appendixes

8.1 THE FINNISH NATIONAL GRID COORDINATE SYSTEM

8.1.1 Introduction

In this text the horizontal coordinate system used in Finland is briefly described. Also the transformation parameters between the EUREF89 (WGS84)-coordinate system and Finnish horizontal coordinate system are introduced. The parameters can be used for instance in navigational purposes to transform "GPS"-positions into the Finnish coordinate system.

8.1.2 The reference ellipsoid and the map projection

On maps the true shape of the earth (mathematically described as a reference ellipsoid) is commonly represented as a plane on which every point of the earth's surface is projected. There are several ways to do that. Earth's surface can be directly projected on to a plane (azimuthally projection) or first on to a cone or on to a cylinder, which then is opened make a plane (conic or cylindrical projection). Common to all of these projections is that the true shape of the earth can be only approximately represented as a plane.
The Finnish Geodetic Institute started triangulation in Finland in 1919 and the Hayford ellipsoid (dimensions determined by Hayford 1909) was selected as a reference ellipsoid. This ellipsoid is also known as International ellipsoid 1924. The dimensions of that ellipsoid are

- $a = 6378388.0$ m and
- $f = 1/297.0$.

The Transverse Mercator projection with Gauss-Krüger grid was selected for mapping purposes in 1922 (referred to as Gauss-Krüger projection in this text). Gauss-Krüger projection is a transverse cylindrical projection (the axis of the cylinder is at right angles to the axis of the earth). The projection is done in the following way:

- The equator is the straight side line of the cylinder and the y-axis of the horizontal coordinate system.
- The cylinder touches the globe along a great circle (the central meridian). On the plane the central meridian is a straight line and it's length is right (scale = 1.0). The central meridian is the x-axis of the horizontal coordinate system.
- The other meridians are not straight lines but slightly curved lines which intersect the central meridian on the pole.
- The parallels represented as slightly curved lines are orthogonal to the meridians.
- The scale, shape, area and bearing are all projected with very little, if any, distortion.

- The scale error due to the round shape of the earth is the greater the greater the distance from the central meridian is. The effect of that error is reduced using narrow projection zones (3° wide in Gauss-Krüger projection).

8.1.3 Coordinate systems

8.1.3.1 The horizontal coordinate system

In Finland the basis for determining the horizontal coordinates is the National Grid Coordinate System, in Finnish it is called Kartastokoordinaattijärjestelmä (kkj). The Basic coordinate system (Peruskoordinaatisto) and the Uniform coordinate system (Yhtenäiskoordinaatisto) based on that grid system are represented on the topographic maps.
The kkj replaced the older national grid system known as vvj in 1970. After the Finnish Geodetic Institute has carried out the observations of the first order triangulation network of Finland, the network was adjusted using one fixed point (Common Adjustment of 1966). The European Datum 1950 (ED50) coordinates of that particular point were held fixed. In that way the coordinates of the first order triangulation network of Finland could be introduced in the approximate ED50-system. The differences between those coordinates (transformed to the Gauss-Krüger projection plane) and the vvj-coordinates were big. To avoid problems that might have occurred because of that difference on already printed topographic maps those horizontal ED50-coordinates were transformed to match better the older vvj-system. Transformation parameters were determined between horizontal coordinates of the ED50-system and the vvj-system. Using those parameters the horizontal ED50-coordinates were transformed. Those transformed coordinates defined the new kkj-system.
In the Basic coordinate system Finland is divided to four projection zones of 3° wide. The corresponding central meridians are 21°E (zone 1), 24°E (zone 2), 27°E (zone 3) and 30°E (zone 4). The grid of the basic coordinate system is directed along the central meridian. The origin is the intersection of the equator and the central meridian. Thus the x-coordinate (or northing) is distance from the equator and y-coordinate (or easting) is distance from the central meridian. To avoid negative values for the y-coordinates the y-coordinate of the central meriadian is 500 000 m. And furthermore, y-coordinates of the different zones are distinguished by adding the ordinal number of the particular zone before the actual coordinate value. So the values of the y-coordinates of the central meridians are 21°E = 1500 km, 24°E = 2500 km, 27°E = 3500 km and 30°E = 4500 km.
In the Uniform coordinate system Finland is represented in one projection zone. The central meridian is 27°E. Other details equal the Basic coordinate system.
On topographic maps the Basic coordinate system is represented in black colour and the Uniform coordinate system in red colour. In the third zone (27°E) coordinate systems are equal and markings are in red colour.

8.1.3.2 Elevation system

There have been used many national elevation systems in Finland. As the precision levellings proceeded the older elevation system was replaced by a newer one. The present Finnish elevation system is called N60 elevation system.

The zero level of N60-system coincides the mean level of the sea surface in Helsinki in 1960. The N60 system based on the second precision levelling, which was completed in 1963. Since 1968 the new N60 elevation system was used in national mapping activities. However, in Lapland, which is the most northern part of Finland, the temporal elevation system was replaced by the N60 elevation system in 1977. On topographic maps the elevations are represented in the N60 elevation system.

8.1.4 *Transformations between the international EUREF89*-coordinate system and the Finnish national grid system

After the GPS-measurements of a nationwide network in 1992 it became possible to determine the transformation parameters between EUREF89 reference system and the national coordinate systems. The EUREF89 system differs little from the WGS84-coordinate system which is the reference frame of the Global Positioning System (GPS). Because the bench marks of the GPS-network have also known coordinates in ED50 and in kkj-systems it was possible to transform coordinates from one system to another. In this text it is described two different ways to do it.

8.1.4.1 The two-dimensional transformation EUREF89 - kkj

First the EUREF89 (WGS84)-coordinates have to be transformed to the Gauss-Krüger projection plane (zone 3 (27°E)). This can be done using the formulas derived by R.A. Hirvonen (see the references). Next step is to transform those horizontal EUREF89-coordinates to the kkj-system. Transformation equations are

$$\begin{bmatrix} x_{kkj} \\ y_{kkj} \end{bmatrix} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix} \bullet \begin{bmatrix} x_{EUREF} \\ y_{EUREF} \end{bmatrix} + \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix}$$

Values of the parameters are

a = 1.0000021251,
b = 0.0000044173,
ΔX = 132.858 m and
ΔY = 132.490 m.

Note that the y-coordinate has to be in complete form including the zone number.

8.1.4.2 Three-dimensional transformation EUREF89 - ED50 + two-dimensional transformation ED50 - kkj

The second way to transform EUREF89 (WGS84)-coordinates to kkj-coordinates has to be done in two steps. First the three-dimensional EUREF89-coordinates (X,Y,Z) have to be transformed to ED50-coordinates. Secondly those ED50-coordinates have to be transformed to horizontal coordinates of the Gauss-Krüger projection to the zone 3 (27°E). Finally the two-dimensional transformation can be performed. Equations for the three-dimensional transformation are

$$\begin{bmatrix} X_{ED50} \\ Y_{ED50} \\ Z_{ED50} \end{bmatrix} = (1+m) \bullet \begin{bmatrix} 1 & \varepsilon_Z & -\varepsilon_Y \\ -\varepsilon_Z & 1 & \varepsilon_X \\ \varepsilon_Y & -\varepsilon_X & 1 \end{bmatrix} \bullet \begin{bmatrix} X_{EUREF} \\ Y_{EUREF} \\ Z_{EUREF} \end{bmatrix} +$$

where m is scale correction,

ε_X, ε_Y, ε_Z are the rotations about X,Y,Z-axes and

ΔX, ΔY, ΔZ are the shifts.

Values of the parameters are

$$\begin{aligned} \Delta X &= 93.477 \text{ m}, \\ \Delta Y &= 103.453 \text{ m}, \\ \Delta Z &= 123.431 \text{ m}, \\ \varepsilon_X &= -0''.246, \\ \varepsilon_Y &= 0''.109, \\ \varepsilon_Z &= 0''.068 \text{ ja} \\ m &= -2.062 \text{ ppm.} \end{aligned}$$

The horizontal coordinates (ED50-system) can be transformed to kkj- system using following equations.

$$\begin{bmatrix} x_{kkj} \\ y_{kkj} \end{bmatrix} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix} \bullet \begin{bmatrix} x_{ED50} \\ y_{ED50} \end{bmatrix} + \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix}$$

Values of the parameters are

a = 1.0000007515,
b = 0.0000043933,
ΔX = -61.5805 m ja
ΔY 95.6691 m.

Note that in this case the y-coordinate has to be without the zone number.

8.1.4.3 Elevatios

The ellipsoidal height of the EUREF89 (WGS84)-system can be transformed to the N60 elevation system using the Geoid model computed by the Nordic Geodetic Commission (NKG). Transformation equation is

$$H = h - N,$$

where H is the orthometric (levelled) height (N60-system),
h is the ellipsoidal height (EUREF89 (WGS84)-system) and
N is the geoidal undulation (from the NKG geoid model).

If the geoid model is not available, the approximate orthometric height (N60) can be achieved by subtracting 18 m from the ellipsoidal height.

References

This text based mainly on the Atlas of Finland (folio 112: Mapping of Finland). The transformation equations in chapter 4.2 are from a publication of the Finnish Geodetic Institute (Matti Ollikainen: GPS-koordinaattien muuntaminen kartastokoordinaattijärjestelmään (in Finnish)).

R.A. Hirvonen: Die Gauss-Krügersche Projektion fur breite Meridianstreifen auf dem Internationalen Ellipsoide. Suomen Geodeettisen laitoksen julkaisuja, no. 36, Helsinki (publication of the Finnish Geodetic Institute).

Example: a coordinate transformation example

In this chapter the EUREF89-coordinates of one triangulation point is transformed to the kkj-system. Both of the transformations are used. Because the transformation parameters are defined for the hole country, the accuracy of the kkj-coordinates is not very good. However, for navigational purposes those equations are better than they need to be.

Ellipsoidal coordinates (EUREF89): phi 63°46'04.7071
lambda 27°38'30.5830
height 245.909 m

1. Transformation (chapter 4.1)	2. Transformation (chapter 4.2)
Horizontal EUREF89-coordinates (Gauss-Krüger projection plane, zone 3 (27°))	Ellipsoidal coordinates (ED50)
x = 7074147.997 m	phi 63°46'05.2353
y = 3531665.550 m	lambda 27°38'33.7001
Horizontal kkj-coordinates	Horizontal coordinates (ED50)
x = 7074280.288 m	x = 7074338.863 m

y = 3531836.794 m y = 531709.713 m

Horizontal kkj-coordinates

x = 7074280.263 m
y = 531836.861 m

The true kkj-coordinates are:
x = 7074280.405 m
y = 3531837.668 m

3. Height transformation

Ellipsoidal height (EUREF89) : 245.909 m
Geoidal undulation (NKG) : 17.417 m

Orthometric height (N60) : 228.492 m (true levelled height = 227.91 m)

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